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Modal Analysis of Beams Using Finite Element Methods

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***Abstract*—In this development, a modal analysis of free-free and fixed-free beams undergoing free vibration is investigated in Ansys APDL (15.0) based on Finite Element Analysis (FEA) and analytically using Euler-Bernoulli beam theory. Three different materials including: Steel, Aluminum, and Copper are considered in this study. The numerical results obtained from FEA using Ansys are compared with the analytical solution based on Euler-Bernoulli beam formulation and a good agreement is found with a correlation factor (R^2) greater than 0.99. In addition, the natural frequencies for the free-free beam are found to be much higher than those of the cantilever beam and the natural frequencies enlarge by increasing the mode number in both of the cases.**

Keywords—modal analysis, beams, vibration, FEM, Ansys

I. INTRODUCTION

Beams are major structural elements used in different applications such buildings, bridges, railways, robotics arms, trucks and others. Because beams are important structural elements, vibrational analysis of beams has been a vital task in the design of beam for researchers and engineers for more than a century [1], [2]. Expensive work related to vibration of beam has been reported in the literature [3], [4]. In [5] free vibration analysis of F3S.20S aluminum composite material cantilever beam through Ansys APDL is proposed. By taking a fixed material and cross-section area, it has been found that the frequency decreases with increase in length of the beam, however, the decreasing effect of frequency is more rapid in higher mode shapes compared to lower in lower mode shapes. Simulation of refractories resonance frequency is investigated in [6], it was concluded that, the simulation of resonance frequencies can support the evaluation of laboratory experiments by predicting the Eigen modes, and the oscillation amplitude in each sample point can be simulated for arbitrary location and direction of excitation which facilitates the selection of a hammer impact point for the laboratory investigations. In the area of smart materials and structures, vibration of beam is commonly used in energy harvesting applications using piezoelectric and/or other smart materials [7]. Modeling and design of composite free-free beam piezoelectric resonators is introduced in [8] where they presented a model for the analysis of asymmetric composite free–free beam piezoelectric resonators and fabricated a PZT based devices to validate the model. A good agreement is obtained among the experimental results and finite element analysis with the composite model predictions for the resonance frequencies of both free–free beam elements and torsional anchors.

In [9] theoretical modal analysis of freely and simply supported RC slabs is given. The dynamic characteristics of freely and simply supported RC slabs have been analyzed by theoretical modal analysis and the effect of boundary conditions on natural frequencies and mode shapes of the slabs was the main focus. By comparing numerical and analytical natural frequencies, a good correlation was obtained between the numerical and analytical results. It was found that simply support boundary condition has obvious influence to increase the natural frequencies and change the mode shape of slabs compared to freely supported boundary condition.

The analysis of static and free vibrations of functionally graded material (FGM) beams using Timoshenko's beam theory considering a unified approach that integrates both shear deformation and rotational effects has been introduced in [10]. It was concluded that, utilizing Timoshenko's theory for the static and dynamic analysis of FGM beams provides a robust framework for understanding their complex behavior, crucial for engineering applications where material properties are tailored for specific performance requirements.

The comparative analysis of the vibrational behavior of various beam models with different foundation designs have investigated the effect of different support conditions and material properties on the dynamic response of beams [11]. Overall, the analysis provides insights that can guide the selection of appropriate beam models and foundation designs in engineering applications to optimize performance. A review of vibration analysis and its applications is given in [12]. Other applications of beam vibration include damage detection and identification are also detailed in the literature [13], [14].

In this paper, a modal analysis of free-free and fixed-free beams is investigated analytically and in Ansys Software [15]. Three different materials were chosen and the obtained result from simulation were compared with analytical results where excellent agreement was achieved. The remainder of this paper is structured as follows: Section II presents material properties and dimensions of the beams, while section III presents the analytical results. Details and

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procedures of the numerical solution are addressed in section IV. The obtained results are presented and discussed in section V, whilst section IV is devoted for conclusion of the article.

II. DIMENSIONS AND MATERIAL PROPERTIES

Modal analysis of three different material (Steel, Aluminum, and Copper) is carried out in Ansys (15.0) environment and analytically using Euler-Bernoulli beam theory. The dimensions of the beam are: $L = 140mm, b = 25mm, and\ h = 12.5mm$ [6] and the material properties are summarized in following table.

TABLE I. MATERIAL PROPERTIES

Properties	Material		
	Steel	*Aluminum*	*Copper*
$E\ (GPa)$	207	69	110
ν	0.3	0.33	0.34
$\rho\ (kg/m^3)$	8050	2700	8960

III. ANALYTICAL SOLUTION

From Euler equation of beams [16] or, the elementary theory of bending of beams (thin beam theory, or Euler-Bernoulli theory), the natural frequencies of the beam under later vibration are found from the relation:

$$\omega_n = \beta_n^2 \sqrt{\frac{EI}{\rho}} = (\beta_n l)^2 \sqrt{\frac{EI}{\rho l^4}} \qquad (1)$$

Where ρ is the mass per unit length of the beam, and β_n is a number depends on the boundary conditions of the problem. The following table list the numerical values of $(\beta_n l)^2$ for typical beam configurations [16].

TABLE II. NUMERICAL VALUES OF $(\beta_n l)^2$ FOR DIFFERENT BOUNDARY CONDITIONS

Beam Configuration	Modes		
	$(\beta_1 l)^2$ *Fundamental*	$(\beta_2 l)^2$ *Second Mode*	$(\beta_2 l)^2$ *Third Mode*
Simply-supported	9.87	39.5	88.9
Cantilever	3.52	22.0	61.7
Free-free	22.4	61.7	121.0
Clamped-clamped	22.4	61.7	121.0
Clamped-hinged	15.4	50.0	104.0
Hinged-free	0	15.4	50.0

The obtained results from the analytical solution for four modes [17] is provided in following tables:

TABLE III. EIGEN-MODES FOR FREE-FREE BEAM (ANALYTICAL)

Eigen-mode	Frequency (Hz)		
	Steel	*Aluminum*	*Copper*
$\sqrt{\frac{EI}{\rho l^4}}$	933.58	930.7	645
	$\omega(Hz)$	ω(Hz)	$\omega(Hz)$
1	3228	3318	2300
2	9168	9139	6334
3	17979	17923	12423
4	29717	29625	20531

TABLE IV. EIGEN-MODES FOR FIXED-FREE BEAM (ANALYTICAL)

Eigen-mode	Frequency (Hz)		
	Steel	*Aluminum*	*Copper*
$\sqrt{\frac{EI}{\rho l^4}}$	933.58	930.7	645
	$\omega(Hz)$	ω(Hz)	$\omega(Hz)$
1	523	521	361
2	3269	3259	2258
3	9168	9139	6334
4	17979	17923	12421

IV. NUMERICAL SOLUTION

Numerical simulations were carried out in Ansys (15.0) based on FEA. In such analysis the material domain of the object-based model is converted to analytical model that is suitable for numerical analysis, in sense of the so-called discretization. In other word, the material continuum was divided into small, small elements by introducing a grid of lines connected by nodes. The beam dimensions, boundary conditions, and properties are defined in each case, free-free beam and for the cantilever beam. The suitable element type utilized is Plane 182 (Quad four nodes) which is selected initially to define the beam cross section (2D element), then, the cross section was extruded in the third dimension for which the element Solid 185 (brick 8 nodes) was specified. Then, by selection appropriate mesh size the beam is meshed, and modal analysis in solution stage is selected. Twelve modes are extracted and expanded in this analysis.

V. RESULTS AND DISCUSSIONS

As mentioned earlier, 12 modes were extracted in this analysis. The following tables list the obtained results for the two boundary conditions (free-free, and fixed-free) for each of the materials.

TABLE V. EIGEN-MODES FOR FREE-FREE BEAM

Eigen-mode	Frequency (Hz)			
	Steel	Aluminum	Copper	
1	3239.9	3229.6	2238.4	Out of plane flexure
2	6028.1	6005.2	4161.3	In plane flexure
3	8367.1	8247.3	5695.0	Torsion
4	8561.1	8531.9	5912.9	Out of plane flexure
5	14496	14422	9989.4	In plane flexure
6	15898	15838	10975	Out of plane flexure
7	16803	16566	11440	Torsion
8	18089	18028	12494	Longitudinal

9	24664	24510	16970	In of plane flexure
10	24699	24593	17039	Out of plane flexure
11	25371	25022	17282	Torsion
12	34125	33671	23259	Torsion

TABLE VI. EIGEN- MODES FOR CANTILEVER BEAM

Eigen-mode	Frequency (Hz)			
	Steel	Aluminum	Copper	
1	523.98	523.39	363.03	Out of plane flexure
2	1026.3	1024.3	710.27	In plane flexure
3	3173.7	3168.1	2197.0	Out of plane flexure
4	4260.0	4200.7	2901.1	Torsion
5	5689.3	5666.8	3926.7	In plane flexure
6	8472.9	8451.9	5859.5	Out of plane flexure
7	9094.0	9075.0	6292.2	Longitudinal
8	12824	12647	8735.2	Torsion
9	13865	13787	9548.0	In plane flexure
10	15637	15585	10802	Out of plane flexure
11	21514	21224	14660	Torsion
12	23489	23325	16146	In plane flexure

As shown, these modes are combinations of out of/in plane bending, torsion, and axial (longitudinal) vibration. Some of these modes are shown below in Fig. 1 through Fig. 4.

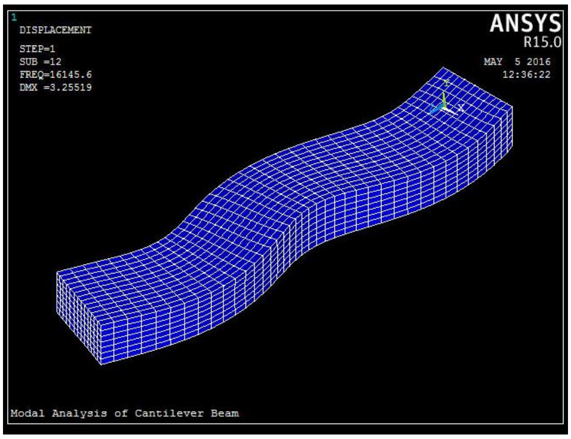

Fig. 1. Meshed beam in Ansys.

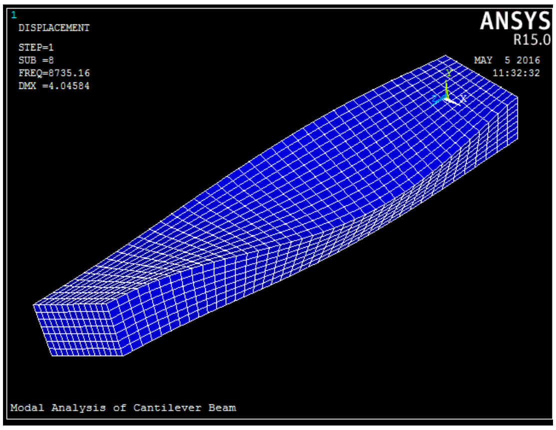

Fig. 2. Torsion Mode.

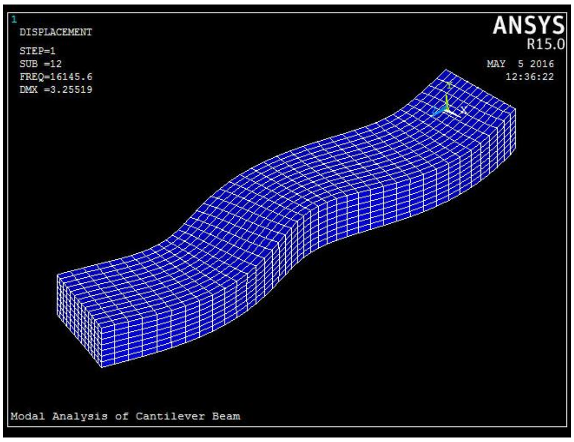

Fig. 3. In plane bending.

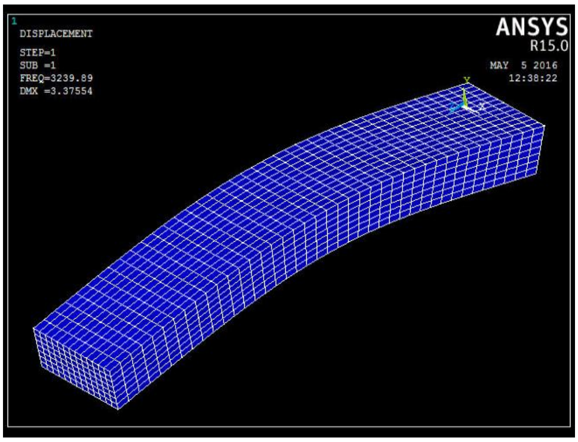

Fig. 4. Out of plane bending.

VI. ANALYTICAL VERSUS NUMERICAL RESULTS

This section present a comparison between the analytical results obtained from the theory of lateral vibration of beam and the numerical results generated from the FEA simulations conducted on the two cases of support for the beam under investigation. The first 4 Eigen-modes, obtained earlier in the analytical results section are plotted versus the corresponding numerical results observed from Ansys. As it is clear from the following figures (Fig. 5. through Fig. 10), the correlation factor is more than 0.99 in all cases, which establish the correctness of the FEA.

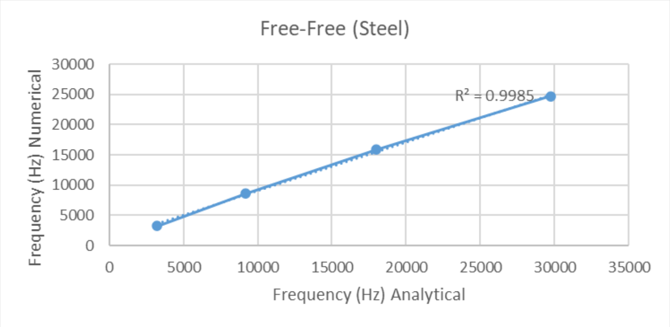

Fig. 5. First 4 modes for Free-Free steel beam analytical Vs numerical.

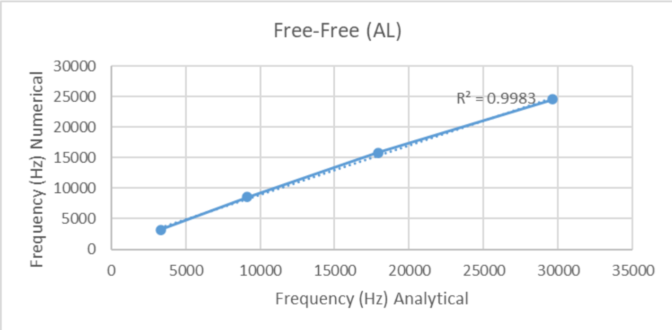

Fig. 6. First 4 modes for Free-Free AL beam analytical Vs numerical.

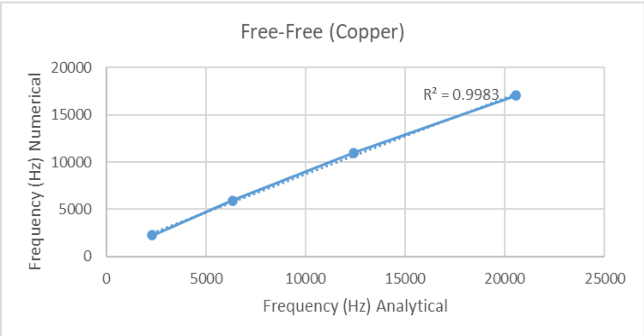

Fig. 7. First 4 modes for Free-Free copper beam analytical Vs numerical.

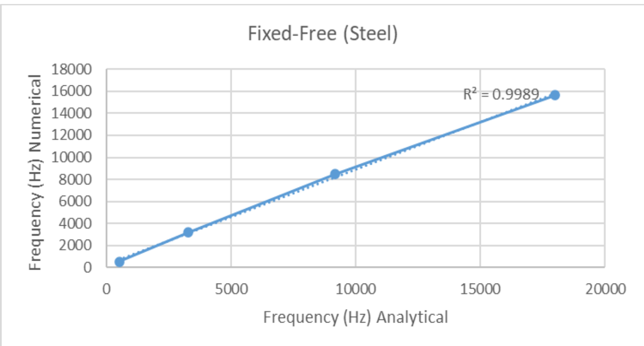

Fig. 8. First 4 modes for Fixed-Free steel beam analytical Vs numerical.

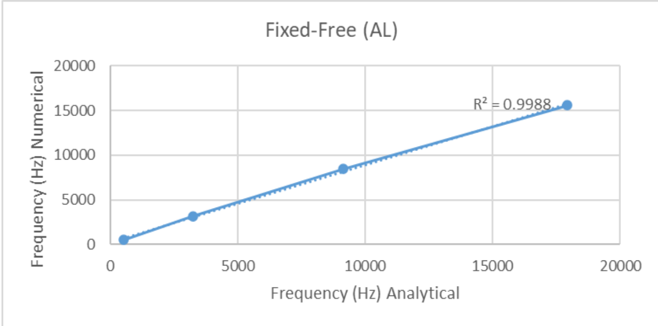

Fig. 9. First 4 modes for Fixed-Free AL beam analytical Vs numerical.

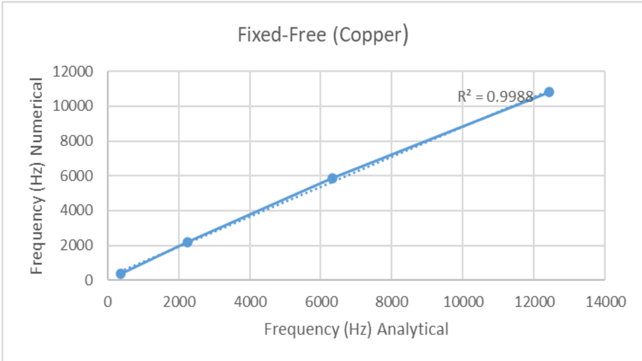

Fig. 10. First 4 modes for Fixed-Free Copper beam analytical Vs numerical.

It's worthy mention that at this point, aside from the fundamental mode, the Eigen-modes obtained from Ansys are not in the same order as the analytical results, and the corresponding frequencies need to be carefully selected. It is found that, the corresponding natural frequencies from the numerical solution are those associated with the out of plane bending only, thus enough modes must be extracted and expanded from Ansys such that we ensure to get the first four out of plane flexure modes. In this case under investigation, twelve modes turn out to be sufficient. One other interesting observation is that, the Eigen modes in the free beam and cantilever beam are not following the same trend, but rather, they are different, and this is expected because we have different boundary conditions in the two scenarios, thus, each beam vibrates according to the restrictions associated. Regarding the natural frequencies, it is clear that, the natural frequencies increase by increasing the mode number; and the natural frequencies for the free-free beam are much higher than those of the cantilever beam with the same dimensions and material properties which is obvious, a free beam will vibrate greater than a constrained beam.

VII. Conclusions

In this study, modal analysis of free-free and fixed-free beams is investigated analytically and numerically using finite element analysis (FEA) using Ansys APDL software package for three different materials. Excellent agreement between the numerical and analytical solutions is achieved. The natural frequencies for the free-free beam are found to be much higher than their cantilever beam counterparts and the natural frequencies grow by increasing the mode number in both of the cases. A further extension of this work could be achieved by performing experimental work on such beams and compare the experimental results with the analytical and numerical ones.

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